# Design Strategy for Noise Reducing Particle Dampers on Hydraulic Pumps

**Michael Lenz, Peter Zuppke, Ahmed Shorbagy, and Prof. Jürgen Weber**

*Institute of Mechatronic Engineering, Technische Universität Dresden, Helmholtzstrasse 7a, 01069 Dresden, E-mail: michael.g.e.lenz@tu-dresden.de, fluidtronik@mailbox.tu-dresden.de*

## Abstract

Noise stands as a pivotal design criterion for displacement units, since it can limit their operability in a human environment. Particle damping promises a means of broadband mitigation of structural vibration and associated noise emissions. Cavities are introduced into the vibrating structure and partially filled with particles. Under oscillation, movement of the particles is induced, resulting in partially plastic impacts and friction, thus dissipating vibration energy. The damping effect is determined by a multitude of parameters; at the same time, simulation methods are still very limited in scope. Therefore, preliminary experimental studies are required for the targeted deduction of an integrated pump design.
To perform a flexible assessment, attachable particle damper components were designed and applied to an axial piston pump. The paper details the design process, manufacture, and the experimental setup and results of the parameter study.

**Keywords**. axial piston pumps, noise reduction, NVH, particle damping, FEA, experimental dynamics.

## 1. INTRODUCTION

Noise emission of machines is a determining factor for their acceptance with users and authorising bodies, as exposure to excessive noise causes annoyance and potential health hazards.

For hydraulic drives in particular, recent shifts in the industry are causing increased attention towards their acoustic properties. While classically hydraulic machines often had their noise masked by the louder combustion engines driving them, with electrification the hydraulic unit is often a primary cause of sound emission and vibration excitation. An ever present demand to increase power density carries with it the hazard of noise emissions increasing accordingly, while decentralized electrohydraulic drives make pump noise emission spots ubiquitous. Not only do hydraulic pumps emit significant airborne noise themselves, the transmission of their operating vibration into adjacent components through structure borne noise may cause these to emit even higher levels of noise, especially if thin walled structures such as found in vehicle bodies are affected. The attenuation of structural vibrations of pumps and their associated sound emissions is therefore a central aspect of strategies to reduce hydraulic drive noise.

The intense vibration of displacement pumps has its origin in the periodic pressure build-up and suction processes which are inherent to their working principles. The pump drive can be optimized to reduce oil pressure pulsations and the consequential excitation of vibration, see e.g. [1], [2], [3]. However, this approach can affect pump performance and efficiency negatively, which is often not acceptable.

Active oscillation suppression either on the side of fluid [4] or structure borne noise [5] can provide substantial improvements, but may involve complex mechanisms and requires steering energy. In contrast, passive measures of structure borne noise suppression are typically achieved with rather simple components and require no energy input. The classical passive approach in machine dynamics is oscillation decoupling by resilient transmission elements. While the use of elastic hoses instead of rigid lines is a widespread application of this principle, the pump drive itself requires a high degree of rigidity for proper function. Further typical passive measures include insulation, barriers and mass-tuned dampers; however these come with the drawback of requiring significant additional installation space.

An emerging passive means of vibration attenuation is offered by particle damping. Figure 1 shows the basic setup of a particle damper, comprised of a cavity with solid walls which is filled with loose particles. The cavity may be attached to a vibrating structure as indicated in the figure, or integrated into it. When vibration is induced, its energy is partially transmitted to cause movement of the particles. This movement involves friction and partially plastic impacts of the particles with each other and the walls, upon which vibration energy is dissipated, leading to the damping effect. Damping by this method is effective over wide ranges of vibration frequency and amplitudes [6], [7], acting in arbitrary directions.

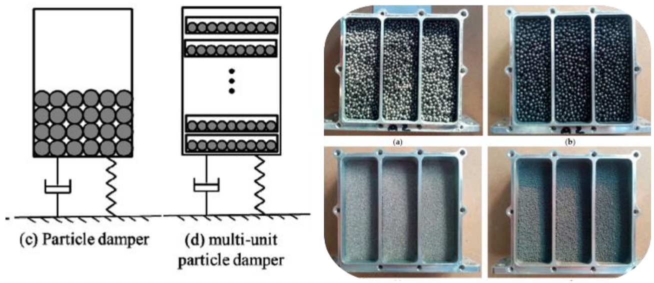



Figure 1: Principle of particle damping, from **[8]**

There is ample literature on the study of particle dampers in laboratory environments or with simulation tools, part of which will be discussed in Section 3. Reports on practical applications remain relatively rare, but often show significant success of the approach in reducing unwanted vibrations, [9], [10].

In the course of a research project, a modified design of an axial piston pump is being developed which incorporates cavities for particle damping into its structural parts. In the following, we present a preliminary study performed with attachable 3D printed particle damper casings. In a first section, an analysis of operational vibrations of the reference pump is reported. the next section details the design strategy for the dampers, based on this analysis in correspondence with literature on particle dampers. In the final section, the experimental parameter study is described and results are reported.

## 2. OPERATIONAL VIBRATION MEASUREMENT AND FEA

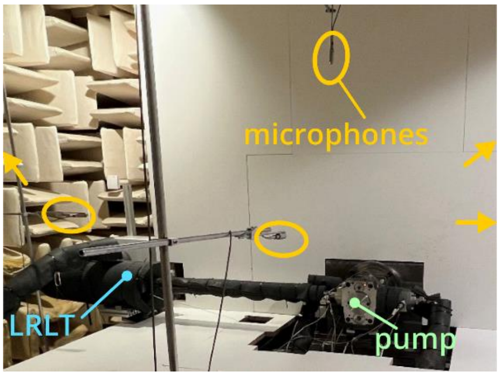



Figure 2: Sound power measurement setup in anechoic chamber

In order to design targeted noise reduction measures, the vibroacoustic behaviour of the pump in operation was studied. The pump was installed on a hydraulic test rig in an anechoic chamber as shown in Figure 2. The emitted sound power was measured at varying combinations of speed, working pressure and swivel angle according to Table 1. Also measured were surface accelerations and pressure pulsation in the low and high pressure lines and a trigger signal for the shaft rotation. The sound power measurement was carried out in an anechoic chamber according to the provisions set out in ISO 3744 / DIN 45635-1 and 26, using 6 microphones on a cuboid enclosing surface limited by 2 reflecting planes. A low reflection line terminator (LRLT) was used in order to suppress reflections of high pressure pulsations from the downstream hydraulic circuit.

Table 1: Operating points for reference measurements

| | **Revolutions per minute** | **Working pressure / bar** | **Relative swiveling / %** |
|---|---|---|---|
| **Sound power measurement (all combinations)** | 500; 1000, 1250, … 2000 | 50, 100, … 300 | 25, 50, 75, 100 |
| **Operational vibration measurement** | 1250 | 200 | 100 |
| | 1500 | 50, 200, 300 | 100 |
| | 1750 | 200 | 100 |

An in-depth assessment of the vibration was performed by means of an operational vibration measurement. For that, one hundred measurement points were defined, with four of them on the supporting flange and the others on the pump surface. At selected operating points (see Table 1), the stationary vibration at each measurement point was captured by a triaxial accelerometer. The sensor was mounted on a magnetic base to allow of fast and precise change of the measurement position. The measurement setup is shown in Figure 3, left.

After synchronization and Fourier transformation it is possible to give a phase-correct vector of complex acceleration amplitude $\vec{A}(f) = \vec{A}_{\mathrm{Re}} + j\vec{A}_{\mathrm{Im}}$, from which vectors $\vec{V}(f) = \frac{1}{j2\pi f}\,\vec{A}$ and $\vec{U}(f) = \frac{1}{j2\pi f}\,\vec{V}$ for surface velocity and displacement can be computed. From the vectors at all measurement points, an operational vibration shape can be deduced for each frequency, as shown in Figure 3, center.

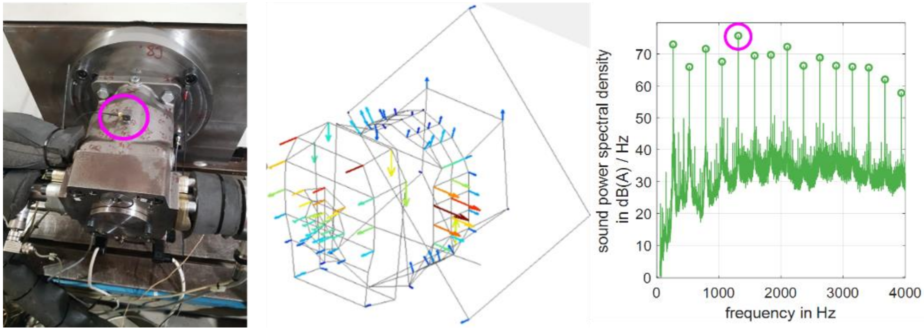



Figure 3: Operational vibration measurement setup, measured vibration shape at critical frequency, for 1750 rpm, 200 bar, and corresponding sound power level spectrum

The Fourier spectrum of A-weighted sound power is used to identify frequencies of critical sound emission. Due to the well-known characteristics of the vibration inducing pressure pulsation in axial piston pumps, the sound power is concentrated in several sharp peaks at frequencies $f_i = i \cdot n \cdot n_{\mathrm{p}}$, with $n$ the revolution speed and $n_{\mathrm{p}}$ the number of pistons, as evidenced by the level spectrum shown on the right graph in Figure 3. Considering the logarithmic nature of the level value, there is little contribution from frequencies in between the peaks. It is concluded that vibrations at a few discrete frequencies cause most relevant sound emission at a given operating point, and targeted damping of these is therefore a promising means of reducing pump noise.

Consequently, the vibration shapes at those frequencies which contribute the most to sound power were evaluated different operating points, in order to determine locations of high normal surface velocity amplitude. Those were found to mainly fall in two groups: on the one hand, low frequency (300-1200 Hz) yaw, pitch and torsion oscillations of the whole pump body relative to the mounting flange, and on the other hand bending modes of the pump casing walls at higher frequencies (2-4 kHz). In the intermediate frequency range, a mix of both is observed. In the example shown in Figure 5, casing wall bending along with a pitching motion of the end case relative to the casing compose the vibration shape.

To support the analysis of operating vibrations, modal analysis of the pump in the test rig was performed both experimentally with the impulse hammer method, and by finite element analysis (FEA). For the latter, a basic model was used, with less detail compared to more advanced studies such as [11]. It doesn't account for hydraulic line admittances, contacts and joints of the lubricated gaps in the drive unit are defined with a simple frictionless condition, and a rigid boundary condition was set for the mounting points. This was deemed sufficient for the purpose of providing a point of reference for the operational vibration measurement, rather than precise quantitative predictions.

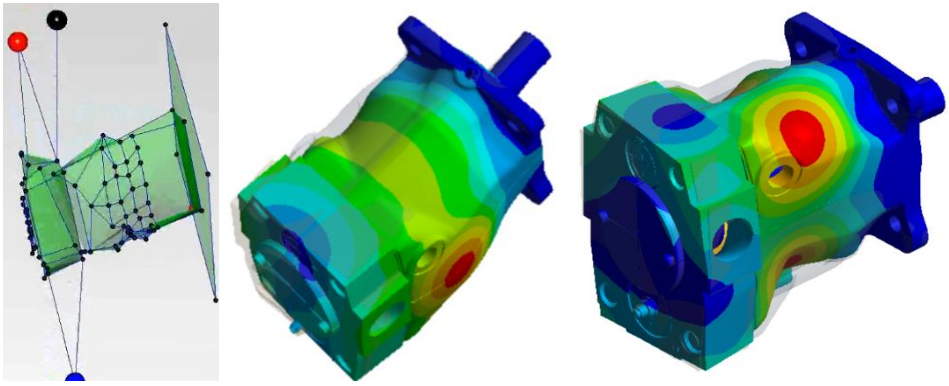


Figure 4: Modal analysis of pump mounted on test rig. Left: top view on yaw mode at 420 Hz, experimental. Middle and right: Whole pump bending at 2.3 kHz, and curved walls bending at 3.0 kHz, FEA.

The analysis found yaw and pitch modes at 3-400 Hz, a torsion mode at 1.2 kHz, and the first mode of elastic deformation of the casing, a bending mode of the pump as a whole, at 2.3 kHz. Further bending modes were observed at subsequent frequencies, first involving the lateral casing walls and subsequently the upper and lower curved walls with modes starting from 3.0 kHz. Experimentally and numerically determined natural frequencies were in accordance of 10% or better, which was deemed sufficient for this qualitative study.

The conclusions drawn from the presented experimental study will inform the placement of damper cavities in the following section.

## 3. Attachable Particle Damper Design And Manufacture

As was shown in Section 2, the pump noise emission, assuming variable speed operation, is driven by vibrations on a wide frequency range from a few hundred up to 4000 Hz. From reports in literature, particle dampers offer a promising means to attenuate such broad-band oscillation. The design of particle dampers poses challenges however:

- There are as of now relatively few industrial applications; most studies concern either idealized laboratory experiments, or simulations, as noted in [12]. Therefore, there is limited experience to draw upon.
- Simulation of particle dampers typically relies on discrete element method (DEM) models, which at present have prohibitive computational costs for practical numbers of particles, especially when finer particles are used. Calibrated concentrated or continuum models as in [7] are application-specific and cannot be generalized.
- There is a multitude of factors influencing particle damper performance, a non-exhaustive overview of which is shown in Figure 5.

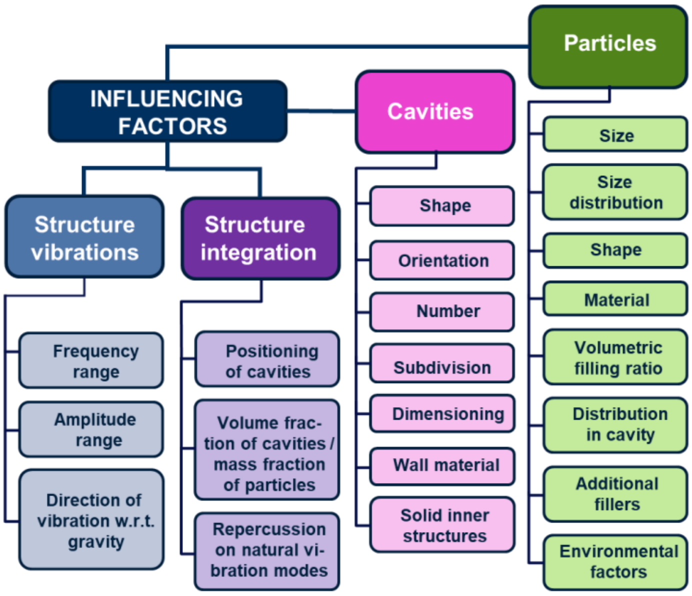



Figure 5: Parameters influencing particle damper performance

It was therefore decided to conduct a preliminary experimental study with the aim of exploring different configurations of the parameters from Figure 5 before embarking on a design for cast pump parts with integrated particle dampers,.

For that purpose, casings were designed and fabricated in 3D polymer print that could easily be attached to on a regular design pump. The casings can be filled with particles on the test rig, allowing for quick changes of both particle types or casings. The study aimed to provide insights for an integrated damper design; additionally, if the particle dampers proved to significantly reduce vibrations, these attachable units could also serve as retrofit solutions for noise reduction in existing pumps.

In a first step, the available literature on particle dampers both in laboratory settings and in practical applications was reviewed in order to deduce construction guidelines. The results are compiled in Table 2.

Table 2: particle damper construction guidelines from literature

| Topic | Source | Finding |
|---|---|---|
| Cavity / structure volume ratio | [7] | Increasing damping performance with increasing volume ratio of the particle damper cavities relative to the overall structure up to a value of 10 %; beyond that rapidly diminishing returns |
| Particle size | [13] | Smaller particle size results in high damping effectiveness over a wider range of amplitudes |
| Particle material | [13] | Best results with tungsten particles, followed by steel; in contrast, polypropylene particles exhibit minimal damping |
| | [14] | Rubber particles perform well at relatively low frequencies;lead granulate has higher damping than steel chips and balls, tested for frequencies up to 350 Hz |
| Filling ratio | [15], [16] | Low filling ratios are beneficial at low frequencies, higher ratios at high frequencies |
| | [16] | Optimal filling ratio between 67 % (at 75 Hz) and 90 % (at 1500 Hz) |
| | [17] | Optimal filling ratio may reach values up to 98% |
| Cavity size | [18] | Higher damping at increasing dimensioning of the cavities in the direction of vibration |
| | [18] | Increasing damping effect with increasing cavity size in the vertical direction up to 30 mm, above that a slight decrease due to the effect of gravity on the particles |
| Cavity geometry | [19] | With spherical cavities, a more broadband damping effect is achieved as compared to a cubic shape, while peak values are lesser |
| Inner structures | [20], [9] | Damping increases when solid internal structures are included sinside the cavity |

From the results of the operational vibration measurement presented in Section 2, specific areas on the pump surface were identified where significant vibrations occur at various operating points. The local vibration amplitudes at specific frequencies were put in relation to the maps of damping effect over frequency and vibration intensity from [6]. However, these cover only the frequency range up to 1 kHz. [7] documents transfer functions at high frequencies for damped and undamped hollow beams for a variety of configurations, using metallic sintering powder as damping particles. It is noted that these only describe frequency, but not amplitude dependency; still they provide a valuable guideline for the kilohertz frequency range.

Based on the literature survey and measurement results, the following preliminary design decisions were made:

- A target value of 10% of the pump casing and end case material volume was set for the cavities.
- Cavities were placed on the pump casing walls in areas where high amplitudes of bending modes were observed in the experimental study in Section 2, as well as on the end case where the yaw, pitch and torsion modes lead to significant amplitudes.
- The particles according to Table 3 were selected for experimental assessment. Focus was placed on cast iron particles, commonly used as blasting abrasives. These were chosen as a cost-effective alternative to the steel balls frequently mentioned in literature (e.g. [12], [8]). An additional damping effect was expected from the size distribution and rough surface, which [13] and [10] report as beneficial for particle damping, as well as from the material damping properties of cast iron. In addition, steel balls and tin granulate were included in the study, the latter with the aim to evaluate a softer metallic material.
- An ideal cavity dimension of 40x40x40 mm³ was set based on the findings in [6]. In particular, the dimension in the vertical direction should not exceed this value so as to avoid the obstruction of particle movement by overlying layers.

Table 3: Particle selection for preliminary study

| No. | Material | Size/mm |
|---|---|---|
| P1 | Cast iron blasting agent | 1.6-2.25 |
| P2 | | 0.8-1.25 |
| P3 | | 0.2-0.4 |
| P4 | Steel balls | 2 |
| P5 | Tin granulate | 2-4 |

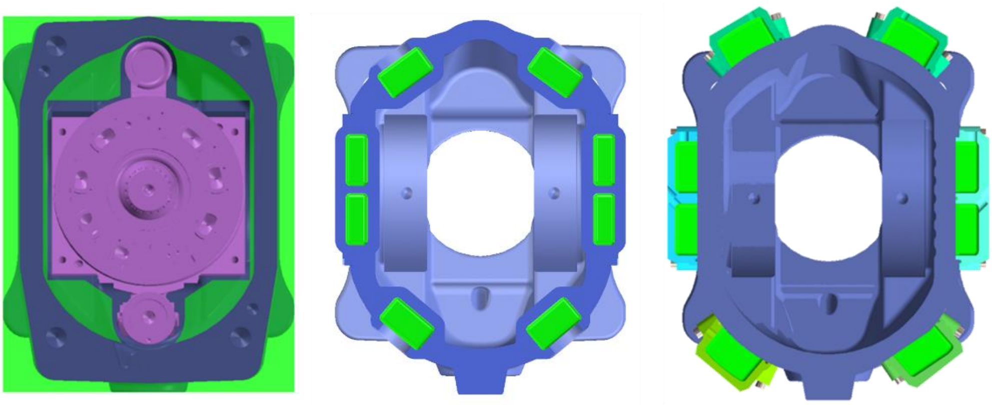


Figure 6: Left: Available construction space. Middle: concept for integrated particle dampers in casing. Right: corresponding design of attachable damper components

The design strategy for damper cavities as detailed above is constrained by practical considerations for pump design. Since the goal is to inspire an integrated design, the dimensions of the attachable dampers need to conform to these restrictions. In particular, it was determined that a significant increase of required space would not be acceptable for an industry adaptation. Therefore, the limits of the bounding box of the existing pumps were not to be exceeded.

A concept for placement of damping cavities in the casing under these constraints was developed, including preliminary sizing of the cavities – the final dimensioning being subject to the outcomes of the experimental study. These cavities were then offset towards the existing pump's surface, and attachable casings were constructed so as to enclose them, as depicted in Figure 6 for the casing; an analogous procedure was applied for the end case. A typical wall thickness of 5 mm was used, which corresponds to estimates for a later cast design. The casings are open towards the vibrating structure without a lid, thus allowing for direct transfer of the vibrational energy into the particle bed.

The final design for the attached dampers is shown in Figure 7. Each casing includes a filling bore for easy insertion of the particles during testing. Sealing cords are used where casings are mounted on raw cast surfaces, to prevent the escape of fine particles. From this base design, variations of the cavities inner structure, stiffening structures, curved surfaces etc. can be deduced for further study. The two pink-coloured casings attached to the front of the end case were included in order to take advantage of the considerable amounts of solid material currently present in these areas, where cavities could potentially be introduced with no additional space needed.

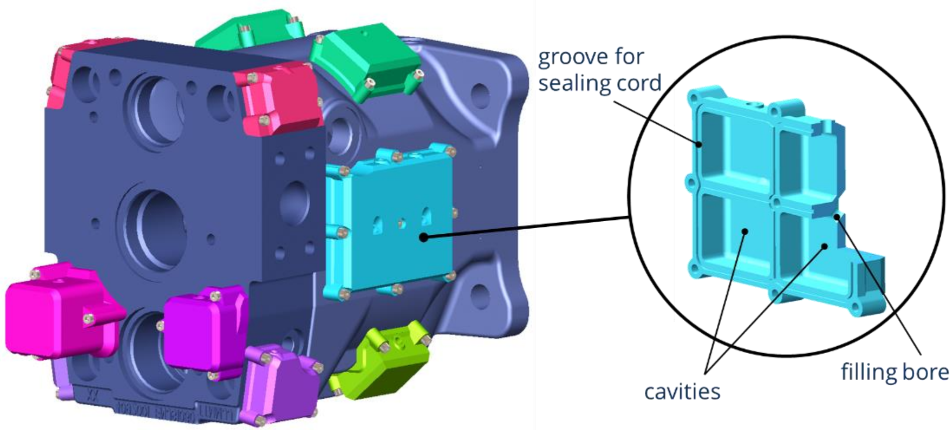



Figure 7: Pump with attached damper components

While the design aims to best simulate the integrated damper concept from Figure 6 (Middle), several differences are observed: there is only one direct interface between the particle bed and the oscillating pump structure; the transfer of momentum through the attachable casing walls may be less due to the transmission path which includes an additional contact and screw connection. The attachable casing material being different from that of the pump structure results in different contact behaviour and hence different transfer of momentum as well. Finally, the integrated cavities affect the pump casing stiffness properties significantly, leading to altered structural dynamic behaviour even before particle damping; meanwhile the attached plastic casings are expected to only minimally affect these properties.

Finally the casings were manufactured on a 3D printer out of polylactic acid (PLA) material. The print was performed with a wall thickness of 1.2 mm and 20% filling in between walls, with reinforcement around the screw bores. This results in a highly lightweight design, reducing the risk of exciting resonant vibrations of the casings, however decoupling of their oscillation results difficult given their low mass. After PLA exhibited significant creep under pump operating temperatures, a second set of casings was printed from polyethylene terephthalate glycol (PET-G). Threaded holes were drilled on the pump casing and end case using a 5-axial CNC machine.

## 4. EXPERIMENTAL STUDY ON PARTICLE DAMPER PARAMETERS

The pump was newly installed on the test rig reported on in Section 2 for the assessment of particle damping parameter combinations. In order to test a wide variety of parameter sets, a limited number of operating points was scheduled, as listed in Table 4, each with one repeat measurement.

Table 4: Operating points for attachable particle damper assessment

| Revolutions per minute | Working pressure / bar | Relative swiveling / % |
|---|---|---|
| 1000, 1250, 1750, 2000 | 200 | 100 |
| 1500 | 100, 150, ... 300 | 100 |

In a first cycle, each particle type listed in Table 3 was to be tested for its. A filling ratio of 80% was set for these tests. Empty cavities would be dismounted. Further measurements were scheduled with all cavities left empty, in order to assess the effect on acoustics of the attachable casings themselves. Also, the performance of the regular – save for the threaded holes – pump without attached parts was measured to establish a baseline for acoustic performance.

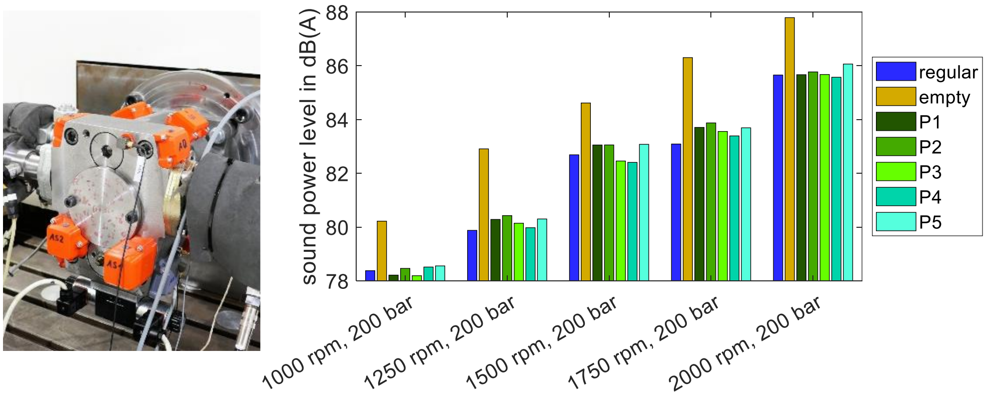



Figure 8: Test rig and performance of plastic made particle dampers depending on particle type (see Table 3 for particle types)

Figure 8 illustrates the experimental setup and the results of speed-dependent sound power level of this first cycle. It is apparent that the empty casings lead to an increase of sound power emission by about 2 dB across all operating points. Filling the cavities largely reverses that effect, returning to similar sound power levels as observed for the regular pump. This is mostly independent of the particle type, even though a tendency is observed for steel balls (P4) to perform slightly better than the others. To further investigate this behaviour, the dependency of the ratio of filling of the cavities was studied with steel balls for particles. The results are depicted in Figure 9. A gradual reduction of sound power is visible. The spectral distribution of sound power provides an explanation for these results: The additional sound power for the empty casings occurs cue to increases above 2.8 kHz, most prominently in the band from 2.8 to 3.5 kHz.

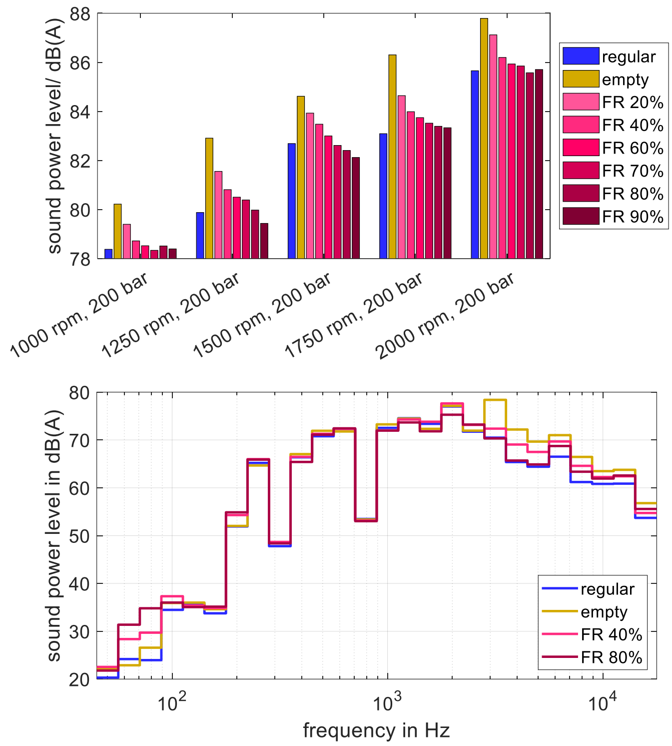



Figure 9: Performance of plastic made particle dampers filled with steel balls depending on cavity filling ratio. Top: integral values of sound power level. Bottom: tertiary band spectrum of sound power level for load case 1500 rpm / 200 bar.

An FEA of the plastic casings shows natural frequencies starting from 2.7 kHz. The analysis makes some simplifying assumptions: walls are modelled homogeneous and a substitute density is set as a quotient of actual weight and model volume, and rigid supports are set as boundary conditions for the nodes on the edges where the screw bores intersect with the contact area. Nevertheless, the result is consistent with the observed increase of sound power being due to resonance phenomena of the casing surfaces in that frequency range. The spectrum also shows these high frequency contributions being damped effectively with increasing cavity filling ratio. It is plausible that a high damping effect is achieved where the casing walls perform high amplitude vibrations due to resonance. However, damping below the regular pump's sound power level occurs only minimally and not systematically across bands. It is also observed that at low frequency bands adding particles increases the emitted sound power measurably is measurably higher than for either the regular pump or the empty casings, even though the contribution to integral sound power is insignificant on the logarithmic scale. This may be explained as a rattling effect of the particles colliding with the cavity walls, which is prominent against the overall very low sound emission of the pump at these frequencies.

Upon review the unsatisfying results obtained with the plastic dampers, the design was revisited for metal casings. On the one hand, with natural frequencies predicted to start in the high 7 kHz range, sound emissions from casing resonances were expected to be much lower and shifted to a frequency range significantly above the one relevant for regular pump noise. On the other hand, it was expected that stiffer casing wall material would improve the momentum transfer between the oscillating structure and the particle bed, in particular at higher frequencies. Due to time constraints, a mix of 7 steel casings additively manufactured by laser powder bed fusion (LPBF), and 5 CNC milled aluminium casings were employed for the experimental study. Figure 11, left shows the test setup with metal casings from both manufacturing variants.

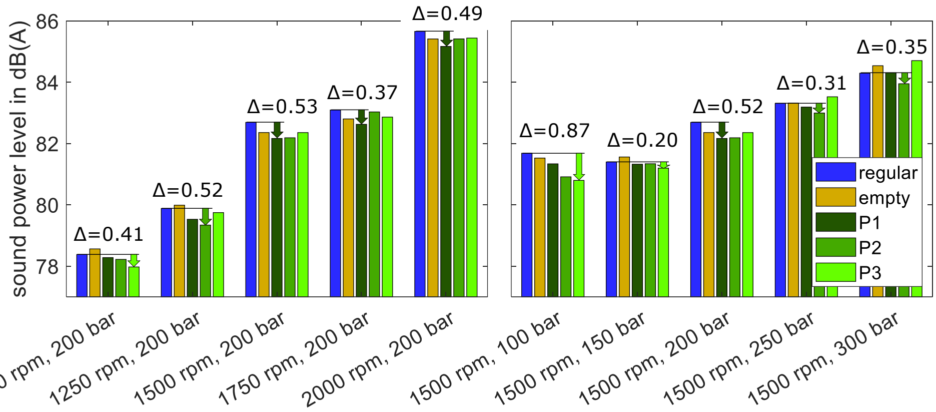
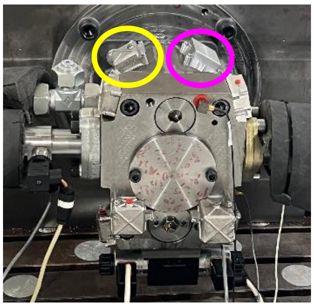



Figure 10: Performance of metal casing particle dampers depending on particle type (see Table 3 for particle types)

The integral sound power level values depending on particle types P1 to P3 (cast iron blasting agent of decreasing size) are shown in Figure 10. Compared with Figure 8, the damping configurations systematically yield lower sound power levels than the regular pump, with the exception of P3 at the two load cases > 200 bar. The reduction is meagre though, with typical values of $\Delta$ = 0.35…0.5 dB for the optimal particle type at each load case. A tendency is observed for the larger particles to perform better at higher pump speeds, and for the fine particles P3 at low power (low speed or pressure). As opposed to the plastic casings, their metal variants don't lead to significant increases of sound power when applied empty, and in fact provide a slight reduction for several load cases.

The spectral representation in Figure 11 shows that the empty casings still cause non-negligible sound emissions in the band starting at 5.6 kHz and above. However, they also slightly reduce the levels in certain bands, leading to an overall lower sound power level for the considered load case. It can be suspected that these reductions are due to oscillation decoupling, such that vibration is not fully transmitted from the pump surface to the cavities at these frequencies. Again, the damping particles are most effective in the frequency bands where the empty casings introduced additional sound emissions.



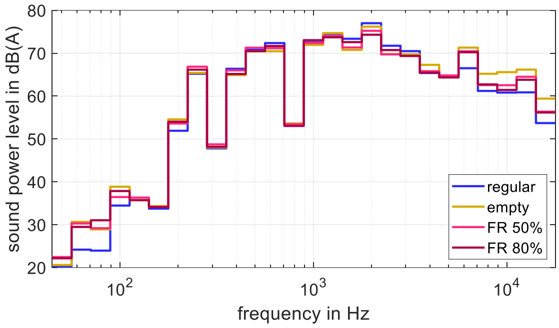



Figure 11: Left: Pump equipped with metal damper casings from additive manufacture (circled yellow) and CNC milling (pink). Right: Tertiary band spectrum of sound power level for load case 1500 rpm / 200 bar for particles P1 depending on filling ratio

## 5. Conclusion and Outlook

Particle dampers for vibration attenuation and consequential noise reduction were fitted to an existing axial piston pump using 3D printed attachable casings. Their design and particle preselection were informed by a literature survey in combination with an experimental and simulative assessment of the pump's vibration behaviour, with operational vibration measurements being the central piece. Experimental assessment of the damping properties showed slight reductions of pump sound power, especially for the revised design with metal casings. The spectral analysis indicates that these reductions are mainly due to oscillation decoupling of the damper casing surfaces, while the particle damping effect is clearly observable only for resonant vibration of the casings themselves.

The presented approach may yield more significant noise reduction when expanded to a larger share of the pump surface, combining a decoupling enclosure strategy with particle damping. The study yielded inconclusive results regarding the viability of integrated campers in the pump casing and further study is required.

## 6. Acknowledgment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