Chapter 6
Experimental Study on Measuring the Constant Off-Set in Displacement from Phase Resonance Testing of Contact-Gap Nonlinearity

Dana Dulcinea Figueroa and Benjamin Robert Pacini

Abstract Phase resonance testing is a popular technique for testing nonlinear structures where closed loop control maintains 90 degrees relative phase between the excitation force and acceleration response. Measurements from these experiments collected at many excitation levels can be used for nonlinear system identification. Typically, accelerometers are used to collect the response data. This data is integrated numerically to produce measured velocities and displacements which are then used to conduct the nonlinear system identification. However, this integration cannot recover certain components of the displacement such as the constant offset typical of contact-gap type nonlinearities. Excluding this data has been shown to produce inaccurate identification of the nonlinearity. This work seeks to demonstrate experimentally the constant offset in displacement as a first step in recovering this information from acceleration measurements. Phase resonance testing is conducted on a structure with a mode which exhibits a contact-gap nonlinearity. Acceleration measurements are acquired via accelerometers, and displacements at various locations on the structure are measured using a high-speed camera. The constant offset in the displacement is shown from the raw measured time histories as well as the error in the displacement obtained via numerical intragrain of the measured acceleration.

Keywords Force appropriation · Nonlinear · Structural dynamics · DC offset · Displacement

Introduction

System identification is an integral task in developing a finite element model and is the process of extracting dynamic properties of a structure from test. For linear structures, this is accomplished via modal testing where natural frequency, damping, and mode shapes are extracted from frequency response functions (FRFs) [1]. For nonlinear structures, system identification is still an open area of research [2]. Of particular interest in this work are asymmetric nonlinearities (such as contact-gap type behavior) where the corresponding restoring forces are not the same in magnitude for positive and negative oscillations of the displacement response. These types of nonlinearities produce constant offsets in the displacement response (referred to as DC offsets in this work) which must be captured either through direct measurement or through post processing to accurately identify the nonlinearity. Acceleration values measured during excitation can be double integrated to get displacement values if and only if the initial conditions of displacement are known. If the initial displacement values are unknown, the DC value will get lost in integration resulting in inaccurate displacement values.

This work is an initial investigation into directly measuring these DC offsets during a force appropriation experiment. Nonlinear force appropriation testing, also known as phase-resonance testing [3, 4], is a technique where closed loop control is used to maintain a structure in phase resonance [5, 6](i.e., 90 degrees relative phase between excitation force and response) as the input is slowly increased. This allows for the characterization of a single nonlinear mode of a structure from low to high energy levels and results in measurements that can be used for system identification.

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The remainder of this paper is outlined as follows: Section 2 provides a motivating example for the importance of properly capturing the DC offset in dynamic testing. Section 3 gives a short background of the previous experimentation from which the data originated. Section 4 covers the data processing for DC extraction. Section 5 describes the processes and equations used to calculate DC offset. Section 6 provides lessons learned, future work to implement the techniques developed in this work, and a conclusion of the entire experimentation and data processing.

Motivation

To demonstrate the importance of including DC offset in the displacement, results from [7] are used where Phase-Resonance Multi-Harmonic Balance (PR-MHB) [8] was used to conduct virtual nonlinear force appropriation tests on a SDOF oscillator with a contact-gap spring element (Fig 1). PR-MHB is a method of solving nonlinear equations of motion in the frequency domain while enforcing constraints which maintain phase resonance between the excitation force and response, so the results are analogous to those produced from a nonlinear force appropriation test. The equation of motion for the contact-gap oscillator is

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + f_{nl}\left(x\left(t\right)\right) = f(t) \tag{1}$$

where f is the excitation force and $f_{nl}\left(x\right)$ is the nonlinear restoring force given by

$$f_{nl}(x(t)) = \begin{cases} 0 & x(t) < x_g \\ k_{\mathrm{p}}\left(x(t) - x_g\right) & x(t) \geq x_g \end{cases} \tag{2}$$

The piece-wise linear penalty spring, k_{p}, activates once the displacement, x, exceeds the gap distance, x_g. Otherwise, the system behaves linearly where the modal properties are defined by mass (m), damping (c), and stiffness (k).

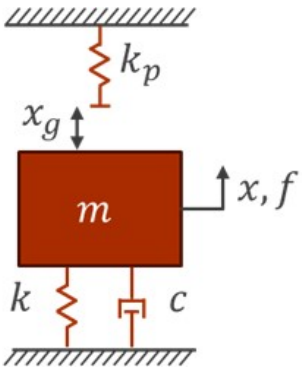

Fig. 1 contact gap oscillator.

The PR-MHB results from [7] of the contact-gap oscillator provide the Fourier Coefficients of the displacement response at the fundamental frequency and associated harmonics as well as the DC component. Thus, $x(t)$ can be reconstructed with and without the DC offset to study the resulting influences on the identification of $f_{nl}\left(x\right)$. In the system identification scenario, it is assumed that a linear modal test has been conducted so that m, c, and k are known. Additionally, the PR-MHB results provide the measured displacement and excitation force from the virtual experiment. Therefore, the "measured" $f_{nl}\left(x\right)$ can be computed by rearranging Eq (1) for cases with and without the DC component in displacement:

$$f_{nl}\left(x\left(t\right)\right) = f\left(t\right) - m\ddot{x}\left(t\right) - c\dot{x}\left(t\right) - kx(t). \tag{3}$$

As mentioned previously, $\ddot{x}\left(t\right)$ is typically the response quantity measured during a force appropriation test and is used to compute $\dot{x}(t)$ and $x\left(t\right)$ numerically via integration. However, the DC offset in displacement is not recovered in this process, so the scenario explored here where the DC offset is excluded replicates the typical measurement case. Using the Force-Response Curve (FRC) technique for system identification from [7](which is similar to approaches from [9, 10]), $f_{nl}\left(x\left(t\right)\right)$ is plotted versus $x\left(t\right)$ to construct $f_{nl}\left(x\right)$ as shown in Figure 2. Comparing these results with Eq (2), the effect of the DC offset in displacement is evident in that when the DC offset is excluded, the resulting $f_{nl}\left(x\right)$ is incorrect. Therefore, in a practical scenario where a structure exhibits similar asymmetric behavior, if the DC offset is not included in the displacement response, the resulting identification will be erroneous. This work describes initial attempts to measure the DC offset directly from nonlinear force appropriation experiments for nonlinear system identification.

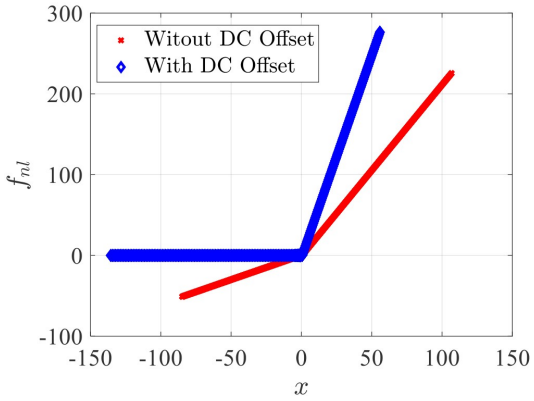

Fig. 2 Nonlinear restoring force for the contact-gap oscillator with and without including the DC offset in displacement.

Test Setup

The data used to extract the DC offset came from a previous experiment done by Alkady et.al [11] in which a half c-beam assembly (Fig 3) was investigated for nonlinear behavior. The purpose of their testing was to address the limitations of traditional sensors used in modal analysis by using full-field, high-speed, and high-fidelity Digital Image Correlation (DIC) with the addition of phase-based motion magnification (PMM)to better characterize the nonlinear response of bolted joints .

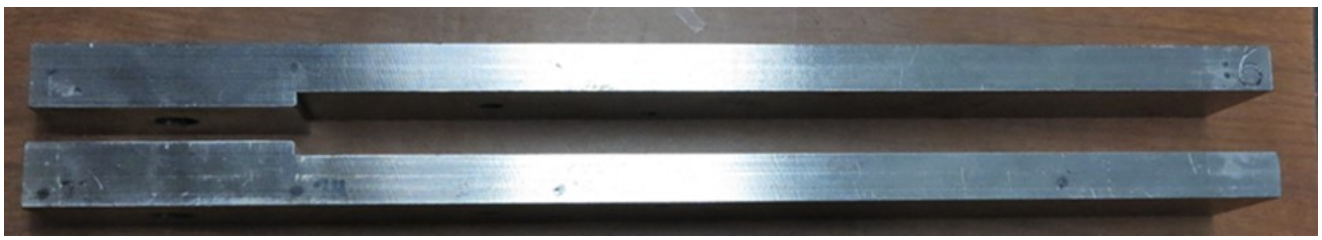

Fig. 3 Unbolted Assembly [11].

The test set up is shown in Figure 4 and consisted of the 2 c-beams (Fig 3) bolted together and suspended by bungee cords to simulate free-free boundary conditions. A speckle pattern sticker was placed at the area of interest, i.e., the joint. A shaker

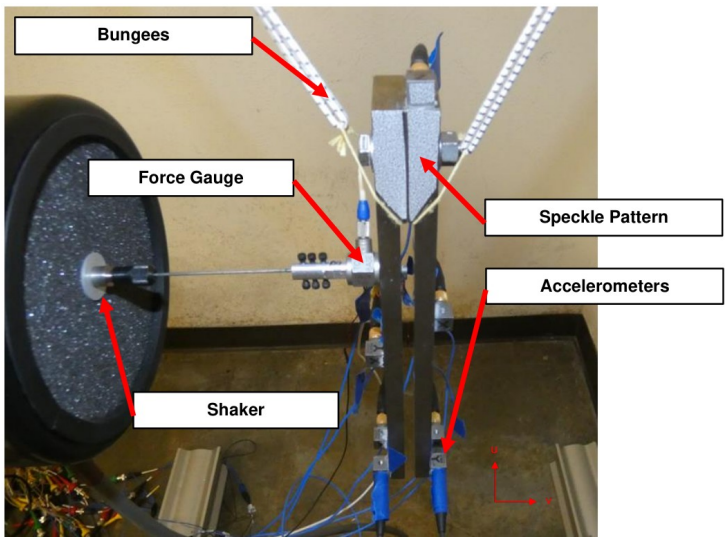

Fig. 4 Test setup [11].

was connected to a location that was optimized for exciting the out-of-phase bending mode which activates open/closing behavior of the joint. High-speed cameras were set up in stereo to get displacements in three dimensions and lights were also placed to properly illuminate the speckle pattern.

Force appropriation using a phase-locking algorithm was used at voltage amplitudes of 0.1V – 0.8V with an increment of 0.1V Before each run, a reference image of the beam was taken at rest for calculation of displacements later. The shaker is then activated at the lowest voltage at a starting frequency suspected to be the desired mode. Phase resonance is found through closed loop control from Siemens LMS. Once phase resonance is found, data is collected for 2 seconds. MatchID, an image analysis software from Siemens, was then used to extract responses from the collected frames of the tests and the displacements of the chosen subsets were calculated. Experimentation showed that as the amplitude of excitation increased, so did the height and length of the gapping along the joint interface which resulted in asymmetric nonlinear behavior.

Data Processing

MatchID was used to align the coordinate system of the beam with that of the master camera. Five sets of nodes were placed near the joint as seen in Figure 5. The v direction data was the focus of this analysis. The variable $v_{ij}(t)$ is the measured v-direction displacement time history of node ij with respect to the reference image. The relative displacement of point pairs across the joint is designated v_j (e.g., $v_5 = v_{15} - v_{25}$).

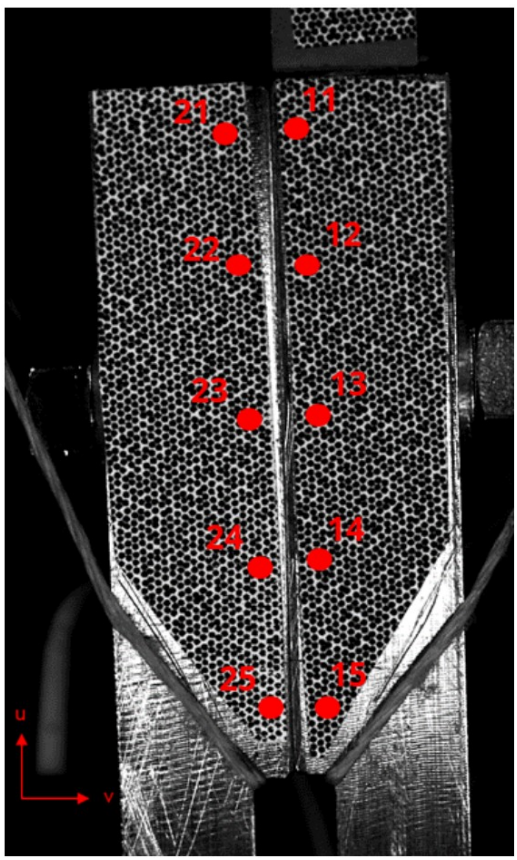

Fig. 5 Approximate point locations.

The objective of this work is to directly measure the DC offset due to nonlinearity, which means we need to extract the frequency content of v_{ij}. This was accomplished by fitting the functional form of a Fourier series (Eq (4)) to the steady state response.

$$v_{ij}(t) = B_0^{ij} + \sum_{k=1}^{N} A_k^{ij} sin(k\omega t) + B_k^{ij} cos(k\omega t) \tag{4}$$

where $v(t)$ is the time history, B_0^{ij} is the DC offset, A_k^{ij} and B_k^{ij} are the of the sine and cosine portions of the k^{th} harmonic, ω is the frequency of the signal, and N is the number of harmonics present. When the measured response is discretized into n samples with $v_{ij}(t_m)$, where $m = 1, \ldots, j$, the following matrix can be constructed:

$$\overline{v}_{ij}(t) = \begin{bmatrix} \sin(\omega t_1) & \sin(2\omega t_1) & \cdots & \sin(N\omega t_1) & \cos(\omega t_1) & \cos(2\omega t_1) & \cdots & \cos(N\omega t_1) & 1 \\ \sin(\omega t_2) & \sin(2\omega t_2) & \cdots & \sin(N\omega t_2) & \cos(\omega t_2) & \cos(2\omega t_2) & \cdots & \cos(N\omega t_2) & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \sin(\omega t_j) & \sin(2\omega t_j) & \cdots & \sin(N\omega t_j) & \cos(\omega t_j) & \cos(2\omega t_j) & \cdots & \cos(N\omega t_j) & 1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \\ B_1 \\ B_2 \\ \vdots \\ B_N \\ B_0 \end{bmatrix} = C\overline{A} \tag{5}$$

The coefficient vector, $\overline{A}$, can be solved for in a least squares sense by

$$\overline{A} = C^{+}\overline{v_{ij}}, \tag{6}$$

where C^+ is the pseudo inverse of matrix C

These coefficients are computed for each DOF at each excitation level, so that their change can be observed from low to high response. This process allows for the extraction of B_0^{ij} from the measured time response v_{ij}. The magnitude of the other harmonics is computed using Equation (7) and is compared to B_0^{ij} to evaluate the significance of the DC offset in the measured displacement [5].

$$M_k^{ij} = \sqrt{{A_k^{ij}}^2 + {B_k^{ij}}^2}, \tag{7}$$

DC Offset Directly from DIC Measurement

Once the magnitude of the harmonics is computed for each DOF using Equation (7), they are plotted against each input voltage level. Figure 6 shows the harmonic content for each point in Figure 5 with even harmonics plotted as dotted lines, odd harmonics plotted as solid lines, and the DC (B_0) plotted as a black line.

There is a non-monotonic trend with respect to voltage in the B_0 for all points. Moreover, the B_0 is largest when the displacement (and therefore the nonlinear response) is the lowest. Since the magnitude of B_0 is commensurate with the strength of the nonlinearity, it is reasonable to expect that the largest B_0 should occur at the largest voltages. however, this is not the trend observed in Figure 6; the largest B_0 occurs at the lowest voltage. Therefore, it appears that a phenomenon is contributing to the B_0 in addition to the nonlinear physics.

It is believed that the other contributor to the measured DC offset is static, rigid translation of the beam within the camera frame. The measured DC offset (B_0) is thus the sum of the DC due to nonlinear dynamics ($B_{0,NL}$) and that due to static motion of the part within the camera frame ($B_{0,Cam}$):

$$B_0 = B_{0,cam} + B_{0,NL} \tag{8}$$

For system identification, $B_{0,NL}$ is desired and therefore $B_{0,Cam}$ needs to be determined. We hypothesize that $B_{0,cam}$ can be determined by finding the time instances when the structure is undeformed. Since the measured values of points v_{ij} are in relation to the reference image, subtracting point pairs directly across the joint from each other removes the effect of the static rigid translation of the part within the camera frame. Therefore, when this difference is equal to 0, the relative position of the point pairs is the same as their relative position in the reference image, and thus represents the time instances of the undeformed state. These time instances could then be used to adjust the measurements of v_{ij} to remove the effect of $B_{0,cam}$. This process is carried out with two point pairs ($v_{21} - v_{11} = \Delta v_1$ and $v_{25} - v_{15} = \Delta v_5$) as shown in Figure 7, where the red x's represent the time instances when Δv_i is equal to zero. It is left as a future work to use this data to adjust the measure v_{ij} to remove the effects of$B_{0,cam}$.

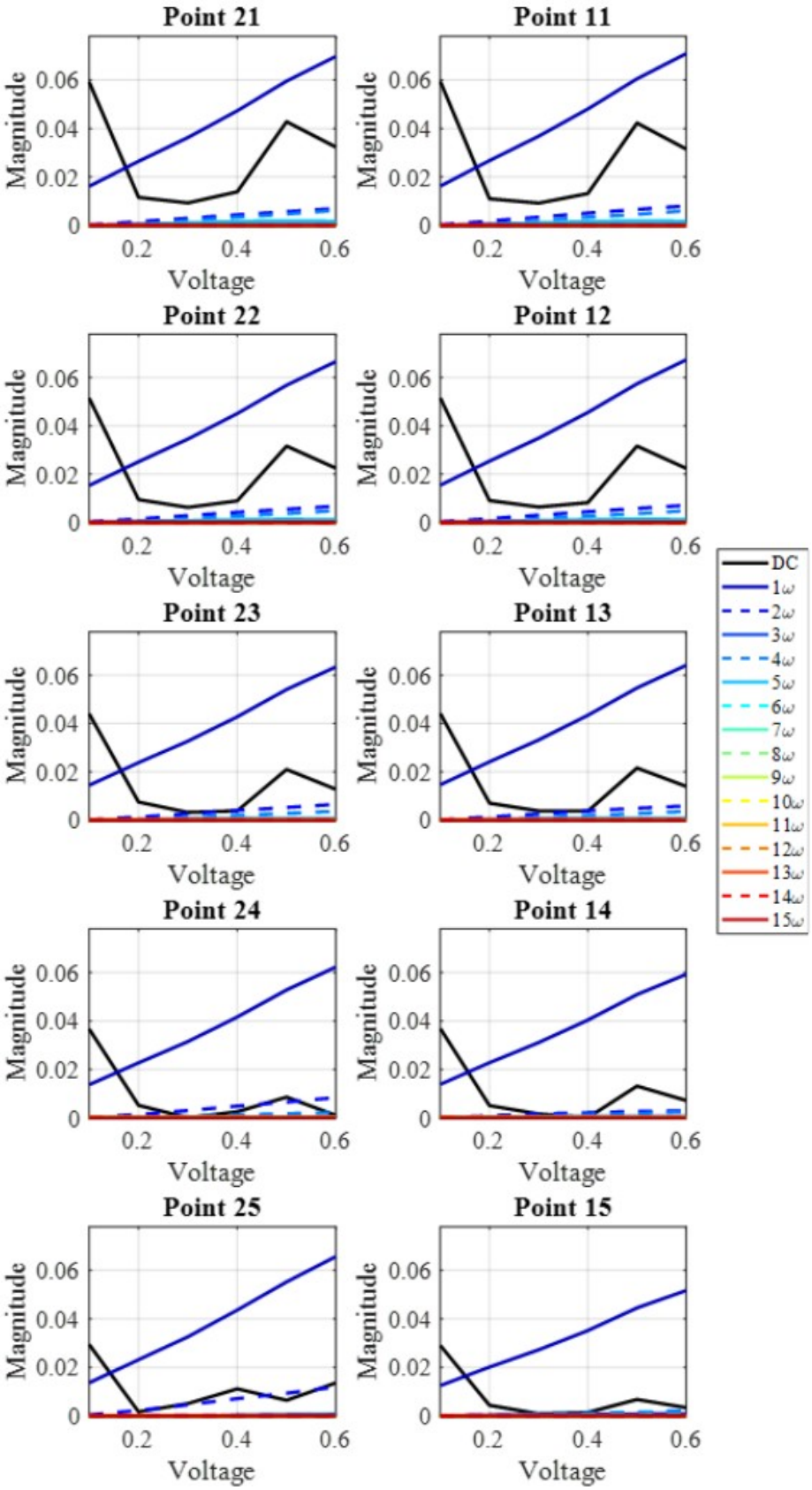

Fig. 6 Harmonic content at each point from Figure 5.

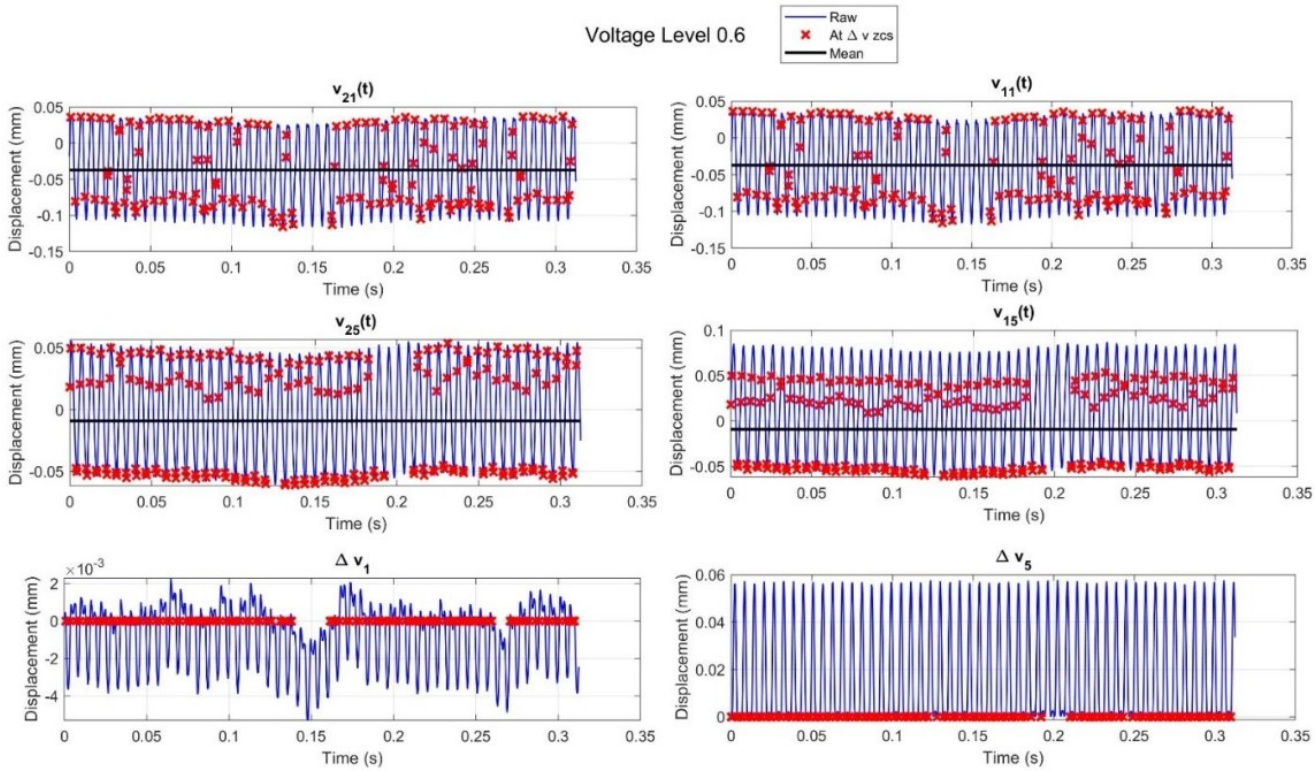

Fig. 7 Plots of the zero-crossings of point sets 1 and 5.

Conclusions

While not all efforts were discussed here, there are many lessons learned from this exploratory work. Three-dimensional displacement measurements obtained using stereo DIC measurements are required since 2D motions measured with a single camera interpret out-of-plane motion as in-plane motion. This was realized during an initial setup (separate from that from [4]) where 2D motion of six points on a full c-beam assembly were measured using a single high-speed camera. While analyzing the results, the in-plane displacements were distorted by out-of-plane motion of the c-beam, leading to the use of data from [4].

This work demonstrated that even with stereo measurements, the extraction of the DC offset due to nonlinear physics from measured response is not straightforward. Static rigid translations of the part within the camera frame contribute to the observed DC offset and must be removed to perform nonlinear system identification. A simple technique was proposed in this work which utilized differences in the displacement of point pairs directly across the joint to determine time instances of when the structure was undeformed. This information could then be used to adjust the measured displacements such that the static, rigid motion of the part within the camera frame is removed. This is left as a future work.

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